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A new guidance technique, referred to as 4-D guidance, is being developed to improve the operation of future STOL aircraft transportation systems. 4-D guidance refers to a technique of synthesizing a complex three-dimensional flight path from simple pilot inputs and flying the aircraft along the path according to an ATC specified time schedule. The two major elements of a 4-D guidance system are the trajectory synthesizer and the control law for flying the aircraft along the synthesized trajectory using the aircraft's autopilot and autothrottle. Inputs to the trajectory synthesizer are the three-dimensional coordinates of waypoints, the turning radius, the speed range, the acceleration limits and the arrival time at time control waypoints. First the three-dimensional trajectory is computed using circular arcs and straight lines. Then the airspeed profile, compensated for wind, is calculated to achieve the desired arrival times. The pilot is informed if the arrival times cannot be achieved. The synthesized trajectory is stored as a time sequence of reference states and controls which the aircraft is forced to track using a linear feedback law.

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4-D GUIDANCE SYSTEM DESIGN WITH APPLICATION TO STOL AIR TRAFFIC CONTROL

PAPER 14-1

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Abstract

A new guidance technique, referred to as 4-D guidance, is being developed to improve the operation of future STOL aircraft transportation systems. 4-D guidance refers to a technique of synthesizing a complex three-dimensional flight path from simple pilot inputs and flying the aircraft along the path according to an ATC specified time schedule. The two major elements of a 4-D guidance system are the trajectory synthesizer and the control law for flying the aircraft along the synthesized trajectory using the aircraft's autopilot and autothrottle. Inputs to the trajectory synthesizer are the three-dimensional coordinates of waypoints, the turning radius, the speed range, the acceleration limits and the arrival time at time control waypoints. First the three-dimensional trajectory is computed using circular arcs and straight lines. Then the airspeed profile, compensated for wind, is calculated to achieve the desired arrival times. The pilot is informed if the arrival times cannot be achieved. The synthesized trajectory is stored as a time sequence of reference states and controls which the aircraft is forced to track using a linear feedback law. On the assumption that 4-D guidance will be available on board future STOL aircraft, a technique is described for assigning conflict free landing times for aircraft arriving in the terminal area.

Introduction

Advanced STOL and VTOL aircraft will soon become important elements in the nation's air transportation network. In order to exploit the unique operational advantages of these aircraft, they must be equipped with guidance and navigation systems that satisfy far more stringent performance requirements than those currently used in conventional aircraft. Performance specifications for these systems are determined primarily by air traffic control constraints, noise abatement maneuvers, and the necessity to fly STOL and VTOL vehicles near obstruction and within very restricted airspace. Noise abatement and restricted airspace corridors require precise guidance and navigation along curved three-dimensional path almost to the touchdown point. This technique is referred to as 3-D guidance. Air traffic control requires the aircraft to follow a specified time schedule along the path. 3-D guidance plus time control along the path is referred to as 4-D guidance.

This paper describes the design of a 4-D guidance system that will be flight tested in a STOL aircraft at NASA Ames Research Center. It includes a discussion of design considerations and an algorithm for trajectory synthesis and control implemented on the airborne computer. Finally, it addresses itself to the question of how an air traffic control center may utilize the precise trajectory control of a 4-D guidance system to sequence and space aircraft for landing.

An essential input to a 4-D guidance system is precision navigation data. The design and flight tests of a navigation system which can provide this data is discussed in the papers by Schmidt, and McGee and Smith in this session.

*On assignment from the U. S. Army

Design Considerations and Interaction with ATC

Chief criteria for selecting the 3-D path are STOL terminal area maneuver requirements and simplicity of computation. These criteria are met by synthesizing the 3-D path from geometrically simple elements. In the horizontal plane these elements consist of segments of circles and straight lines. Complex flight paths are obtained by interconnecting several line segments and sections of circles with different radii. Paths constructed in this manner can yield minimum time trajectories as discussed in Refs. 1, 2, and 3. The vertical trajectory is synthesized from sections of constant flight path angle. The complete three dimensional flight path is then obtained by requiring the aircraft to fly the vertical profile along the ground track determined by the previously computed horizontal trajectory. A critical problem in implementing this synthesis procedure is minimizing pilot work load in entering a trajectory. As explained in the next section, this problem is solved by using waypoints to specify the trajectory.

After the three-dimensional path has been established, the desired position of the aircraft along the path as a function of time is determined from considerations of air traffic control. Generally, more than one aircraft will be flying along the 3-D flight path or will be merging with the path at certain of its points referred to as merging points. This is illustrated in Fig. 1. Aircraft on the two approach routes merge in the vicinity of the approach gate. In the current air traffic control system the final approach controller is responsible for merging aircraft at this point. He does this by observation of the aircraft on the ATC radar and by issuing speed and vectoring instructions to the pilots. The delays, inaccuracies and other limitations of this manual controller - pilot control loop yield a broad envelope of aircraft trajectories between the feeder fix and the gate, shown as the hatched area in Fig. 1. This manual technique can deliver aircraft to the approach gate with a time accuracy of approximately ± 15 sec. It is well known that the time control accuracy influences the required spacing on final approach, which in turn determines the landing capacity of the runway. (4) Through the use of 4-D guidance techniques the accuracy of time control can be greatly increased, thus providing a basis for achieving higher landing rates and greater automation in the control of terminal area traffic. Instead of the controller issuing vectoring and speed commands to space aircraft, the air traffic control system specifies only the desired arrival times at a small number of points along the terminal approach route leaving the burden of computing

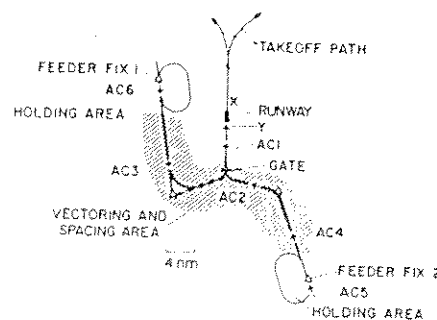


Fig. 1. Typical terminal area route structure.

aircraft control inputs to achieve these times to the on-board system.

From a knowledge of the 3-D path and the desired arrival times at specified points on the path, the 4-D guidance system computes the required airspeed profile. The airspeed computation algorithm must consider the minimum and maximum permissible airspeed, the aircraft's acceleration and deceleration capability, the landing approach speed and the effect of winds. Also, in specifying arrival times air traffic control will use only limited knowledge of aircraft's performance capabilities. Therefore, the algorithm must first determine the feasibility of the specified arrival times by comparing them with the true minimum and maximum times the aircraft can achieve without deviating from the 3-D path. If the specified arrival times cannot be achieved, air traffic control must be requested to reassign arrival times or permit delaying maneuvers such as holding and/or path stretching.

In the preceding discussion of arrival time assignment, the question left unanswered was how air traffic control will generate the arrival times to be assigned to each aircraft. To clarify this question, consider an aircraft equipped with a 4-D guidance system which has just arrived at one of the feeder fixes in Fig. 1. It is assumed that the aircraft had previously been cleared to proceed toward the feeder fix. Those aircraft currently flying between the feeder fixes and the runway were previously assigned arrival times at the gate and the touchdown point. Given the schedules of these aircraft and, for the new aircraft, an estimate of the minimum and maximum times to the gate and from the gate to the touchdown point, a technique is needed to specify feasible times at the gate and at the touchdown point such that separation standards between aircraft are satisfied and the aircraft lands in minimum time. A general algorithm to calculate such times has been described in Ref. 5. This algorithm is briefly reviewed in the last section of this paper.

Specification of 3-D Path

The problem of specifying and calculating the 3-D path is divided into two problems solved in sequence. First the projection of the trajectory in a horizontal plane, is computed from an analysis of waypoint coordinates and desired turning radii. Then the known arc length of the horizontal trajectory together with the altitude difference between adjacent waypoints is used to determine the flight path angle and, therefore, the altitude profile between adjacent waypoints.

A crucial part in the calculation of horizontal trajectory parameters is the interpretation of waypoints. This is explained with the help of an example trajectory shown in Fig. 2. The trajectory begins at liftoff and terminates at touchdown, waypoint 7. Although this trajectory can be flown by a STOL aircraft, its shape has no significance beyond illustrating the construction procedure. All parts of the trajectory consist of segments of straight lines and circles, with values of turning radii given in the figure. The basic problem to be solved can be stated as follows: What is the essential input information that the pilot must provide to the airborne computer in order to generate this trajectory uniquely. The solution lies in the definition of two types of waypoints, which are referred to as ordinary and final heading waypoints.

Ordinary Waypoints

This type is exemplified in Fig. 2 by waypoints 2, 4 and 5. Its location is defined by the intersection of two straight lines. The two lines are connected by an arc of

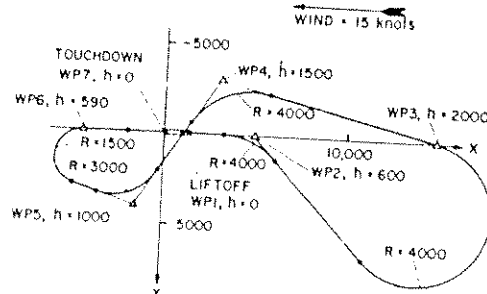


Fig. 2. Example trajectory.

a circle tangent to both lines. Thus, the sharp corner at the waypoint is rounded to obtain a trajectory the aircraft can fly. The radius of the circle used in rounding the corner can either be explicitly specified by the pilot, as in this example, or it can be implicitly determined from a maximum bank angle constraints, ϕ_{max} . For a given ϕ_{max} , the minimum turning radius R_{min} depends on the maximum ground speed V_{gmax} that can be attained in a 360° turn and is

$$R_{min} = \frac{V_{gmax}^2}{g \tan \phi_{max}} \quad (1)$$

where g is the acceleration of gravity. The maximum ground speed is the sum of the maximum airspeed and the magnitude of the wind vector.

Throughout this paper, the runway centered coordinate system is used to specify points in the plane. The origin of this system is at the touchdown point, waypoint 7, the x axis points in the landing direction and the y axis points to the right when facing in the landing direction. Heading angles are measured clockwise from the direction of the positive x axis.

Suppose the pilot has entered the x and y coordinates of a sequence of ordinary waypoints together with the turning radii to be used in rounding the corners at waypoints. From this information the on-board computer calculates various parameters defining the trajectory. It is convenient to compute these parameters successively starting with the last waypoint to be flown through and ending with the first one. For this purpose Fig. 3 shows the trajectory between waypoints $i-1$ and $i+1$, and also defines various quantities used in the calculation. It is assumed that the calculations from waypoints $i+1$ to the last waypoint have been completed. These calculations yielded the quantities i_{i+1} , x_{i+1} , x_{pi+1} , and ψ_{i+1} , which together with x_i , y_i , R_i and x_{i-1} , y_{i-1} , are used to obtain the parameters for the i th waypoint. The heading of the straight line segment between waypoints $i-1$ and i is ψ_i , given by the relation

$$\psi_i = \arctan \frac{y_i - y_{i-1}}{x_i - x_{i-1}} \quad -180^\circ < \psi_i \leq 180^\circ \quad (2)$$

The heading change ψ_{ti} in the circular segment near waypoint i is

$$\psi_{ti} = \frac{\text{mod}}{180^\circ} (\psi_{i+1} - \psi_i), \quad -180^\circ < \psi_{ti} \leq 180^\circ \quad (3)$$

where the direction of the turn is to the right for $\psi_{ti} > 0$ and to the left for $\psi_{ti} < 0$.

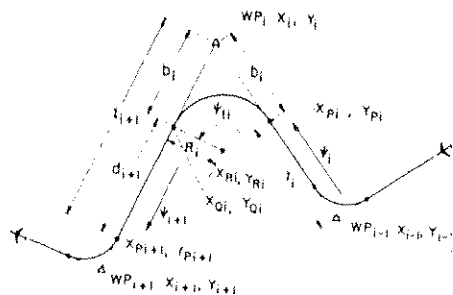


Fig. 3. Trajectory construction for ordinary waypoints.

Next, calculate the quantity b_i shown in Fig. 3.

$$b_i = R_i \tan |\psi_{i+1}| / 2 \quad (4)$$

The length of the straight segment d_{i+1} is then

$$d_{i+1} = l_{i+1} - b_i \cdot l_{i+1} \geq 0 \quad (5)$$

If d_{i+1} from Eq. (5) is less than zero, adjacent turns overlap the computation of the trajectory must stop, and the pilot is given a diagnostic message, such as "waypoints i and $i+1$ are too close." The coordinates for the end of the turn and the center of the turn are

$$x_{Qi} = x_i + b_i \cos \psi_{i+1} \quad (6)$$

$$y_{Qi} = y_i + b_i \sin \psi_{i+1} \quad (7)$$

$$x_{Ri} = x_{Qi} \mp R_i \sin \psi_{i+1} \quad (8)$$

$$y_{Ri} = y_{Qi} \pm R_i \cos \psi_{i+1} \quad (9)$$

where the upper sign is chosen for a right turn and the lower sign for a left turn. Next, the distance l_i is calculated:

$$l_i = \sqrt{(x_{i-1} - x_i)^2 + (y_{i-1} - y_i)^2} - b_i \quad l_i \geq 0 \quad (10)$$

If l_i is less than zero, the calculation cannot continue and the pilot is given a diagnostic message, such as "waypoints $i-1$ and i are too close." If $l_i \geq 0$, the iteration is completed by calculating the coordinates of the beginning of the turn:

$$x_{pi} = x_i - b_i \cos \psi_i \quad (11)$$

$$y_{pi} = y_i - b_i \sin \psi_i \quad (12)$$

These iterations are continued until the first waypoint is reached. Since the waypoints entered do not always yield a flyable trajectory, on-board calculation of the trajectory generally will require a system that permits the pilot to correct errors after the system has issued a diagnostic message.

Final Heading Waypoints

This type is illustrated in Fig. 2 by waypoints 3, 6 and 7. Instead of rounding the corner at the intersection of two lines, the trajectory for this type passes through the waypoint at the instant the turn toward the next waypoint has been completed. Thus, the aircraft begins its flight along the straight line segment exactly over the waypoint. There are two reasons for introducing this type of waypoint. First, it simplifies the specification of some trajectory segments, such as the turn at waypoint 3, which contains more than 180° . Recall that the turn at an ordinary waypoint is limited to less than 180° . Second, this type is required if the arrival time at the waypoint is specified. Specification of arrival time at an ordinary waypoint lacks precision since the waypoint coordinates themselves do not fall on the trajectory. By requiring all aircraft that are merging to fly through a point on the merging path with a common heading, the assignment of arrival times at the merging waypoint can be used to achieve precise spacing of aircraft.

In the specification of a trajectory, ordinary and final heading waypoints can be alternated in arbitrary fashion. The general procedure for calculating the trajectory parameters is illustrated in Fig. 4 with a final heading waypoint, i , embedded between waypoints $i-1$ and $i+1$ of arbitrary type. As before, the trajectory is computed backwards from the last waypoint. The final heading, ψ_{i+1} to be achieved at waypoint i was previously determined in the calculation for waypoint $i+1$ and is therefore a known quantity. It is evident from Fig. 4 that the desired final heading at waypoint i can be achieved with two trajectories, one ending with a left turn, the other with a right turn. The criterion for selection is to choose the one with the shorter path length between waypoints $i-1$ and i . To make the selection, the coordinates of the centers of the two turns are calculated:

$$x_{Ri} = x_i - R_i \sin \psi_{i+1} \quad (13)$$

$$y_{Ri} = y_i + R_i \cos \psi_{i+1} \text{ (right turn)} \quad (14)$$

$$x_{Li} = x_i + R_i \sin \psi_{i+1} \quad (15)$$

$$y_{Li} = y_i - R_i \cos \psi_{i+1} \text{ (left turn)} \quad (16)$$

Then, the distances squared from waypoint $i-1$ to each center are:

$$d_{Ri}^2 = (x_{Ri} - x_{i-1})^2 + (y_{Ri} - y_{i-1})^2 \quad (17)$$

$$d_{Li}^2 = (x_{Li} - x_{i-1})^2 + (y_{Li} - y_{i-1})^2 \quad (18)$$

From the geometry of the construction in Fig. 4, it can be seen that the trajectory with shorter path length also has associated with it the shorter of the two distances d_{Ri} and d_{Li} . Thus, the right turn trajectory is chosen if d_{Ri} is greater than d_{Li} . If the waypoint $i-1$ lies on the line determined by waypoints i and $i+1$, both trajectories have the same length and the direction of the turn, if a turn is required, must be selected on the basis of another criterion. The next step is to determine whether waypoint $i-1$ lies inside or outside the circle used to define the turn. Suppose the right turn was previously selected. Then the waypoint lies inside if $d_{Ri}^2 < R_i^2$ and outside or on the circle if $d_{Ri}^2 \geq R_i^2$. If

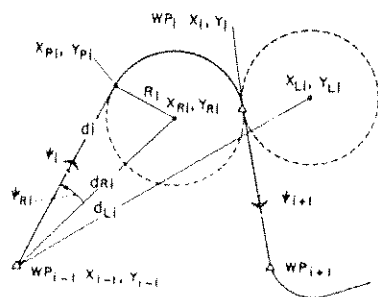


Fig. 4. Trajectory construction for final heading waypoints.

this former case is true the trajectory is not feasible, further computation of the trajectory stops and the pilot is given the message "waypoints i and $i-1$ are too close." In the latter case, the calculation continues with the computation of the heading ψ_{dRi} of the directed line d_{Ri} if a right turn is required or the heading ψ_{dLi} of the directed line d_{Li} if a left turn is required,

$$\psi_{dRi} = \arctan \frac{y_{Ri} - y_{i-1}}{x_{Ri} - x_{i-1}} \quad (\text{right turn}), \quad (19)$$

$$\psi_{dLi} = \arctan \frac{y_{Li} - y_{i-1}}{x_{Li} - x_{i-1}} \quad (\text{left turn}) \quad (20)$$

Next the angle ψ_{Ri} is computed for a right turn

$$\psi_{Ri} = \arcsin \frac{R_i}{d_{Ri}} \quad (\text{right turn}) \quad (21)$$

or ψ_{Li} for a left turn

$$\psi_{Li} = \arcsin \frac{R_i}{d_{Li}} \quad (\text{left turn}) \quad (22)$$

The heading ψ_i is then

$$\psi_i = \frac{\text{mod}}{180^\circ} (\psi_{dRi} - \psi_{Ri}) \quad (23)$$

If the trajectory contains a right turn and

$$\psi_i = \frac{\text{mod}}{180^\circ} (\psi_{dLi} + \psi_{Li}) \quad (24)$$

If it contains a left turn. The length, d_i , of the straight line segment from waypoint $i-1$ to the beginning of the turn is

$$d_i = \sqrt{d_{Ri}^2 - R_i^2} \quad (\text{right turn}), \quad (25)$$

$$d_i = \sqrt{d_{Li}^2 - R_i^2} \quad (\text{left turn}) \quad (26)$$

Finally, the heading change ψ_{ti} in the turn is obtained from the difference between ψ_i and ψ_{i-1} , and the coordinates of the beginning of the turn to waypoint i are

$$x_{pi} = x_{i-1} + d_i \cos \psi_i \quad (27)$$

$$y_{pi} = y_{i-1} + d_i \sin \psi_i \quad (28)$$

Further details, including flow charts for the synthesis procedures are given in Ref. 6.

Technique of Flying to the First Waypoint

In the preceding discussion, the trajectory from the first to the last waypoint was synthesized. To complete the synthesis, the trajectory from the aircraft's initial position and heading to the first waypoint must be constructed.

The construction procedure depends on the type of the first waypoint. If it is an ordinary waypoint, the trajectory is constructed based on the rule that the aircraft will turn from its current heading toward the first waypoint in the direction that minimizes the total path length to the first waypoint. This is the same criterion that was used in constructing the trajectory from waypoint $i+1$ to waypoint i , where waypoint i is of the fixed final heading type. Thus the procedures of the preceding section apply if the initial heading ψ_0 is identified with ψ_{i+1} , the initial position with the coordinates of waypoint i and the coordinates of the first waypoint with those of waypoint $i-1$ and the airplane traverses Fig. 4 in the opposite sense.

If, however, the first waypoint is the fixed final heading type, a procedure different from those discussed so far must be used. This guidance problem can be stated as follows: Determine a trajectory that starts from a given initial position and heading and leads to a position with a specified final heading. Since this problem has been dealt with extensively in Refs. 1, 2, 3, 6 and 7, only a brief discussion of results is given here. References 1 and 2 give minimum time solution to this problem, with Ref. 2 deriving the optimum control law. Simplified solutions are discussed in Refs. 3 and 6. In the simplified treatment, the required trajectory consists of a turn, straight flight, followed by another turn. The parameters of the three segments are chosen so as to satisfy the initial and final conditions of the problem.

Altitude Profile

The calculation of the altitude profile is simple, requiring only the determination of a flight path angle γ_i between waypoints $i-1$ and i . Since the horizontal path length between waypoints is known, the flight path angle is given by

$$\gamma_i = \arctan \frac{h_i - h_{i-1}}{s_i} \quad (29)$$

where s_i is the horizontal path length, between waypoints i and $i-1$ and h_i and h_{i-1} are the altitudes specified at waypoints i and $i-1$ respectively. In order to compute s_i from the segments of turns and straight lines it is necessary to define at what point on the horizontal trajectory the specified waypoint altitude is to be achieved. The rule used is that the waypoint altitude must be achieved exactly at the end of the turn for a given waypoint. This rule is used for both ordinary and final heading waypoints. The last step in the altitude profile computation is to check if each γ_i lies within the range of permissible flight path angles for the aircraft.

It is possible and perhaps desirable to define more complicated altitude profiles between waypoints. Profiles that minimize a performance function such as the fuel consumed could be valuable. However, the limited size of the airborne computer to be used in flight tests of this guidance system, together with the requirement for on-board computation of the trajectory does not permit consideration of more complex techniques at this time.

Speed Profile Computation

Precise time control of aircraft is achieved by determining a feasible speed profile along the already computed

3-D trajectory. A speed profile is said to be feasible if it satisfies the following conditions:

1. The airspeed remains between the minimum and maximum airspeed restriction imposed along the trajectory.
2. The rate of change of airspeed does not exceed the acceleration/deceleration capabilities of the aircraft.
3. The resulting groundspeed yields the desired arrival times at those waypoints where they have been prescribed by ATC (such waypoints are referred to as time-controlled waypoints).

The first two conditions imply that the flying time between any two points on the trajectory is bounded above and below by the minimum and maximum times corresponding to the maximum and minimum airspeeds, respectively. Consequently, if arrival times at the time-controlled waypoints are assigned arbitrarily, then a feasible speed profile may not exist. In order to assure the existence of a feasible speed profile, ATC assigns arrival times based on the minimum and maximum possible flying times between successive pairs of time-controlled waypoints.

The determination of a feasible speed profile in the absence of wind would be a relatively simple task, for in this case airspeed and groundspeed are identical. In the presence of wind, however, the relationship between airspeed and groundspeed is highly nonlinear, and the problem becomes considerably more complex. Assuming a steady wind, the expression for the magnitude of the groundspeed, $V_g(t)$, at any time t is given by:

$$V_g(t) = \sqrt{V_a^2(t) - V_w^2 \sin^2 \psi_w(t) + V_w \cos \psi_w(t)} \quad (30)$$

where $V(t)$ and V_w are the magnitudes of the airspeed and wind speed respectively, and $\psi_w(t)$ is the angle from the wind direction to the ground heading, measured positive clockwise. The differential equation governing $\psi_w(t)$ is:

$$\left| \frac{d\psi_w(t)}{dt} \right| = \begin{cases} 0 & \text{for straight flight} \\ \frac{1}{R} V_g(t) & \text{for circular flight} \end{cases} \quad (31a)$$

$$(31b)$$

where R is the radius of turn. Exact analytic expressions for $\psi_w(t)$ and $V_g(t)$ can be found only in the case of straight flight with constant airspeed. A considerable simplification can be made by assuming that

$$\left(\frac{V_w}{V_a(t)} \right)^2 \ll 1 \text{ for all } t \quad (32)$$

Supporting evidence indicates that inequality (32) is a good approximation not only for CTOL but STOL aircraft operations as well. Under this assumption Eq. (30) can be written as

$$V_g(t) \approx V_a(t) + V_w \cos \psi_w(t) \quad (33)$$

Before considering the speed profiles for straight and curved flight in more detail, it is necessary to establish certain desirable characteristics for airspeed profiles. In the design of the 4-D guidance system described in this paper, the point of view was adopted that the airspeed profile should be a piecewise linear function of time, i.e.,

$$V_a(t) = V_a(t_k) + a_k(t - t_k); t_k \leq t \leq t_{k+1}, k = 0, 1, \dots \quad (34)$$

where a_k is the constant value of acceleration/deceleration in the interval (t_k, t_{k+1}) . Furthermore, changes in airspeed should occur only at a few places along the trajectory, preferably at those points where the minimum and maximum admissible airspeeds change. These requirements were dictated by considerations of passenger comfort, pilot workload, and simplicity of implementation. A typical airspeed profile possessing these characteristics is shown in Fig. 5.

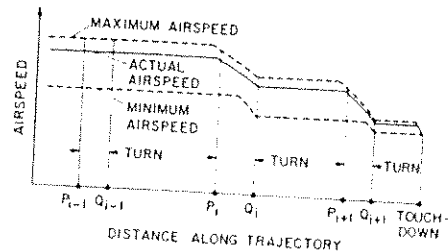


Fig. 5. Typical airspeed profile.

Straight Flight

Let the desired airspeed along a straight flight segment of length d_k be given by Eq. (34). Then the analytic expressions for ψ_w , V_g , and t_{k+1} are

$$\psi_w(t) = \psi_w(t_k) \quad (35)$$

$$V_g(t) = V_a(t_k) + a_k(t - t_k) + V_w \cos \psi_w(t) \quad (36)$$

$$t_{k+1} = t_k + \left[-V_g(t_k) + \sqrt{V_g^2(t_k) + 2a_k d_k} \right] / a_k \quad (37)$$

Curved Flight

Let a curved flight segment consist of a circular turn of ψ_{tk} radians with turning radius R_k ($\psi_{tk} > 0$ for a right turn, $\psi_{tk} < 0$ for a left turn). If the desired airspeed along the circular arc is given by Eq. (34), then no analytic expressions can be found for ψ_w , V_g , and t_{k+1} . Since numerical integration for determining the speed profile would be prohibitive, a different approach is needed. It so happens, that Eq. (31b) becomes integrable if the airspeed has the following form

$$V_a(t) = V_a(t_k) + a_k \left(1 + \frac{V_w \cos \psi_w(t)}{V_a(t_k)} \right) (t - t_k); \quad t_k \leq t \leq t_{k+1} \quad (38)$$

In this case the analytic expressions for ψ_w , V_g , and t_{k+1} are

$$\psi_w(t) = 2 \tan^{-1} \left\{ \frac{C_1}{C_2} \tan \left[\tan^{-1} \left(\frac{C_2}{C_1} \tan \frac{\psi_w(t_k)}{2} \right) + \frac{C_2}{2 R_k \operatorname{sgn} \psi_{tk}} \left(t + \frac{a_k}{2 V_a(t_k)} t^2 \right) \right] \right\} \quad (39)$$

$$V_g(t) = V_a(t_k) + a_k \left(1 + \frac{V_w \cos \psi_w(t)}{V_a(t_k)} \right) (t - t_k) + V_w \cos \psi_w(t)$$

$$t_{k+1} = \frac{V_a(t_k)}{a_k} \left(-1 + \sqrt{1 + \frac{2 a_k}{V_a(t_k)} \bar{t}_{k+1}} \right)$$

where C_1 , C_2 , and \bar{t}_k are defined by

$$C_1 = V_a(t_k) + V_w, \quad C_2 = \sqrt{V_a^2(t_k) - V_w^2}$$

$$\bar{t}_{k+1} = \frac{2 R_k \operatorname{sgn}(\psi_{tk})}{C_2} \left\{ \tan^{-1} \left[\frac{C_2}{C_1} \tan \left(\frac{\psi_w(t_k) + \psi_{tk}}{2} \right) \right] - \tan^{-1} \left[\frac{C_2}{C_1} \tan \left(\frac{\psi_w(t_k)}{2} \right) \right] \right\}$$

(Note that $t_{k+1} = \bar{t}_{k+1}$ if $a_k = 0$). Although V_a given by (38) is not a linear function of time, for values of $V_a(t_k)$ and V_w satisfying inequality (32), $V_a(t)$ turns out to be very nearly linear. This is illustrated in Fig. 6, which shows the airspeed and groundspeed along a portion of the example trajectory of Fig. 2.

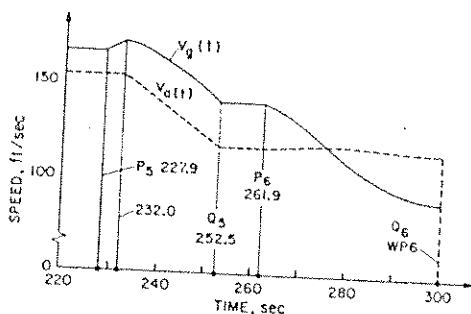


Fig. 6. Portion of airspeed and groundspeed time history for example trajectory, wind = 25 ft/sec.

The only remaining quantity still to be determined is the desired airspeed profile. Since the earliest and latest possible arrival times are achieved by flying the aircraft on the boundaries of the admissible speed ranges, the actual airspeed profile corresponding to an intermediate arrival time must lie between the speed boundaries (see Fig. 5). The nonlinear relationship between arrival time and airspeed necessitates the use of an iterative procedure for the determination of the desired airspeed profile. Basically, the procedure adjusts the cruising speed level between each pair of time-controlled waypoints so that the prescribed arrival times are achieved. This is done by using expressions of the form (37) and (41). A more detailed discussion of the entire speed profile computation can be found in Ref. 6.

A final remark concerning the wind is in order. In this paper only the case of steady wind is considered. It is well known, however, that both the magnitude and direction of the wind are functions of the altitude. If

these functions were known, they could be easily incorporated in the speed profile computation.

4-D Trajectory Synthesis: An Example

The preceding sections described the techniques used to calculate the two major elements of a 4-D trajectory, the 3-D path and the airspeed profile along the path. These elements must now be assembled to produce the complete reference trajectory consisting of the reference states (position, altitude and heading) and the reference controls (turning radius, airspeed and flight path angle) as a function of time from initial to final time.

The reference states and commands are calculated by a procedure which makes use of the chosen parameterization of the 4-D trajectory to minimize computer storage. The calculation is done in two steps. In the first step, the 3-D path and the airspeed acceleration time history along the path are used to construct a command table consisting of a sequence of control inputs arranged in chronological order. Since the reference controls are piecewise constant in time, the command table gives the values of the reference controls only at time instants where they change to new constant values. In the second step, the reference states between command times are computed analytically from the initial condition at the command time and the value of the controls during the command interval. Compared to the technique of storing the reference trajectory at a large number of time instants, this technique uses significantly less storage, an important consideration in implementing the technique on an airborne computer.

The example trajectory shown in Fig. 2 is used to illustrate the technique of 4-D trajectory synthesis described in this paper. The pilot specifies the trajectory to the system by entering the data given in table 1. In the onboard system the waypoint types are replaced by numerical codes. Both, the initial position (lift off) and the touchdown point are treated as final heading waypoints, in synthesizing the trajectory. The initial heading and the runway heading associated with these waypoints are both 0° in this case. Final heading waypoint 6 and the touchdown point are time control points with arrival times of 300 and 350 sec, respectively. The wind is assumed to be from 0° at 25 ft/sec. The airspeed range specified for each waypoint in table 1 is valid from the end of the turn performed at that waypoint to the end of the turn performed at the next waypoint. This requires choosing starting times of decelerations/acceleration such that the airspeed will fall in the next speed range at precisely the end of the turn. These fixed boundary conditions are met by synthesizing the airspeed profile backward from the last waypoint.

The 4-D command sequence generated for this input is given in table 2. The minimum and maximum arrival times at the two time control waypoints (WP6 and WP7) are given at the bottom of the table. There are 17 command times in this example. Columns 3-8 give the states at the command times and columns 9-11 the piecewise constant controls between command times. Space limitations prevent us from giving the equations for computing the reference states between command times. For the same reason the equation for computing the instantaneous bank angle from the airspeed, heading, wind vector and the turning radius is not given. Bank angle and flight path angle commands are applied to the aircraft SAS slightly in advance of those given in the table to minimize errors due to the finite rates of these quantities. Computation time of this trajectory on the IBM 360 is 0.5 sec.

Table 1. Input quantities required for example trajectory of Fig. 2. Initial heading = 0°; runway heading = 0°; airspeed acc/dec = 1.5 ft/sec²; wind speed/direction = 25 ft/sec/0°; initial airspeed = 135 ft/sec.

Waypoint number	Waypoint type	Waypoint coordinates, ft			Turn radius, ft	Airspeed range, ft/sec	Time, sec
		x	y	h			
1	Initial position	1000	0	0	4000	135-135	0
2	Ordinary	5000	0	600	4000	203-304	-
3	Final heading	15000	0	2000	4000	203-304	-
4	Ordinary	3000	-3000	1500	4000	135-203	-
5	Ordinary	1500	4000	1000	3000	110-135	-
6	Final heading	-4500	4000	1000	1500	110-135	300
7	Final heading (touch-down)	0	0	0	1500	110-110	350

Perturbation Equations and Control Law

Having developed a synthesis procedure for the 4-D trajectory, which will now be referred to as the reference trajectory, the next step is to design a control law for flying it. The design of a control law for this problem, which is based on the technique described in Ref. 7, is accomplished by means of a perturbation method. Design of the control law for the altitude channel will not be considered here since this channel is simple with minimal coupling to the other channels.

The nonlinear dynamical equation, from which the perturbation equation are derived, are as follows:

$$\dot{X} = V_a \cos \psi, \quad \dot{Y} = V_a \sin \psi, \quad (42, (43)$$

$$\dot{\psi} = \frac{g(\tan \Phi)}{V_a} \quad (44)$$

where V_a is the airspeed, \dot{X} , \dot{Y} are components of V_a , ψ is the heading, $\dot{\psi}$ the heading rate, Φ the bank angle and g the acceleration of gravity. The wind is assumed to be zero and the flight path angle γ small such that $\cos \gamma \approx 1$.

The perturbation equations are obtained from the nonlinear equations by expansion in a Taylor series about a moving target reference system as illustrated in Fig. 7. The origin and positive x-axis of this system at any given time are the reference position and the direction of the aircraft flying the reference trajectory respectively. In Fig. 7, X_r , Y_r , and ψ_r refer to the reference position and heading and X_a , Y_a , ψ_a to the aircraft position and heading in the runway-centered coordinate system. The linear differential equations obtained from a Taylor series expansion in the perturbed quantities X , Y , ψ shown in Fig. 7 are

$$\dot{X} = V_a + \dot{\psi}_r Y, \quad \dot{Y} = V_r \psi - \dot{\psi}_r X \quad (45)$$

$$\dot{\psi} = g/V_r (\sec^2 \Phi_r) \phi - (\dot{\psi}_r/V_r) V_a \quad (46)$$

where $V_a = V_a - V_r$, $\phi = \psi_a - \psi_r$. The advantage of deriving the perturbation equations in this target referenced system is that terms involving $\sin \psi_r$ and $\cos \psi_r$, which would otherwise appear, are eliminated, thus simplifying the perturbation equations for curved trajectories. Equations 45 and 46 also show that x and y are coupled when $\psi_r \neq 0$, while the gain of the ψ channel is inversely proportional to V_r , so that $\dot{\psi}_r$ and V_r are parameters which depend on the reference trajectory. A control law for nulling the perturbed quantities will therefore have to contain $\dot{\psi}_r$ and V_r as parameters.

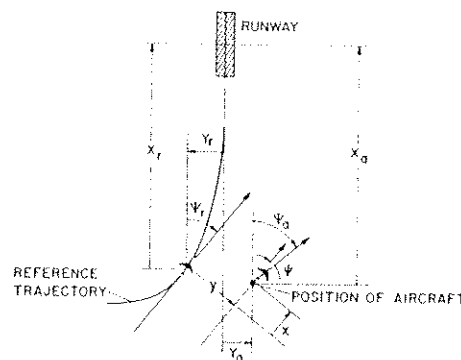


Fig. 7. Moving target reference system.

Table 2. 4-D command sequence for example trajectory

Command sequence number	t, sec	States				Controls				
		x, ft	y, ft	h, ft	ψ , deg	V_g , ft/sec	V_a , ft/sec	\dot{V}_a , ft/sec ²	R, * ft	γ , deg
1	0	1000	0	0	0	110	135.0	1.5	straight	6.2
2	18.2	3249.6	0	243.2	0	137.7	162.3	1.5	4000 r	6.2
3	33.6	6187.8	1285.9	600	47.3	174.1	191.1	1.5	straight	3.6
4	64.1	9383.6	4749.1	898.3	47.3	210.9	227.9	0	straight	3.6
5	76.0	11091.6	6594.7	1057.5	47.3	210.9	227.9	0	4000 l	3.6
6	143.7	15000	0	2000	-166.0	252.1	227.8	0	straight	-2.0
7	159.8	11071.9	-979.4	1860.2	-166.0	252.1	227.8	-1.5	straight	-2.0
:	:	:	:	:	:	:	:	:	:	:
14	300.3	-45000	0	590.0	0	91.1	116.1	1.5	straight	-7.5
15	300.5	-4482.5	0	587.7	0	91.1	116.4	0	straight	-7.5
16	345.4	-378.2	0	49.6	0	91.1	116.4	-1.5	straight	-7.5
17	350.0	0	0	0	0	85.0	110.0	0	straight	0.0

*r = right turn, l = left turn

T_{\min}/T_{\max} to WP6 = 252/324 sec

T_{\min}/T_{\max} to WP7 = 295/376 sec

The control variables of the aircraft for tracking the reference aircraft are the bank angle ϕ_c and the speed V_a . A linear model of the combined autopilot and aircraft dynamical response for a bank angle and velocity command system can be approximated by the following equations,

$$\ddot{\phi} = (1/\tau_\phi) (-\dot{\phi} + k_\phi \phi - k_\phi \phi_c) \quad (47)$$

$$\ddot{V}_a = (1/\tau_v) (-\dot{V}_a + k_v V_a - k_v V_c) \quad (48)$$

where ϕ_c and V_c are the command inputs and ϕ and V_a are the response. The parameters in these two equations, k_ϕ , k_v , τ_ϕ , τ_v were deduced by matching the step responses of these equations to those of a currently in-service 4-engine jet-aircraft with an autopilot and autothrottle. Their numerical values are 0.375 sec⁻¹, 0.167 sec⁻¹, 1.04 sec, and 4.17 sec, respectively. The following control law is chosen for nulling the perturbed quantities x , y and ψ ,

$$\dot{\phi}_c = -k_{\phi y} y - k_{\phi \psi} V_r \dot{\psi}_r x \quad (49)$$

$$\dot{V}_c = -k_{vx} x - k_{vy} \dot{\psi}_r y \quad (50)$$

Note that Eqs. (49) - (50) contain $\dot{\psi}_r$ and V_r as parameters. This parameterization has been found to be effective in achieving acceptable performance of the control law for the class of reference trajectories of interest here.

The governing factors for determining the numerical values of the five gains k_{vx} , k_{vy} , $k_{\phi x}$, $k_{\phi y}$, and $k_{\phi \psi}$

are (a) the accuracy of the navigation data, (b) the allowable bank angle and throttle activity for passenger comfort, and (c) the accuracy of following the synthesized reference trajectory. A root locus analysis of the closed loop system indicates that a good compromise between conflicting requirements (b) and (c) is to use 0.0002 rad/ft for $k_{\phi y}$, 0.004 rad/rad (ft/sec) for $k_{\phi \psi}$, 0.0001 rad/ft (sec/rad) for $k_{\phi x}$, 0.04 (ft/sec)/ft for k_{vx} , and 0.15 (ft/sec)/ft (sec/rad) for k_{vy} . This combination of gain constants was obtained by trial and error using root-locus analysis of Eqs. (45)-(50). The roots corresponding to this set of gains yield reasonable frequency and damping for all values of $\dot{\psi}_r$ from zero to 6°/sec. A root locus plot of Eqs. (45)-(50) as a function of $\dot{\psi}_r$ for the choice of gains given here can be found in Ref. 7.

Simulation Results

Figure 8 shows the block diagram of the simulation used to evaluate the guidance system. The general flow of computations in the trajectory synthesis algorithm are indicated inside the block drawn with dashed lines. The final product of the synthesis computations is the command schedule. The Reference States and Controls Generator uses the command schedule, clocktime, and the measured wind vector to compute the reference states and controls for each control time interval. An interval of 0.1 seconds was used in the simulation. By use of the measured wind vector, ground speed and ground heading are converted to reference airspeed V_r and airspeed heading ψ_r .

The control loop used to fly the aircraft along the reference trajectory is shown in detail. The first step

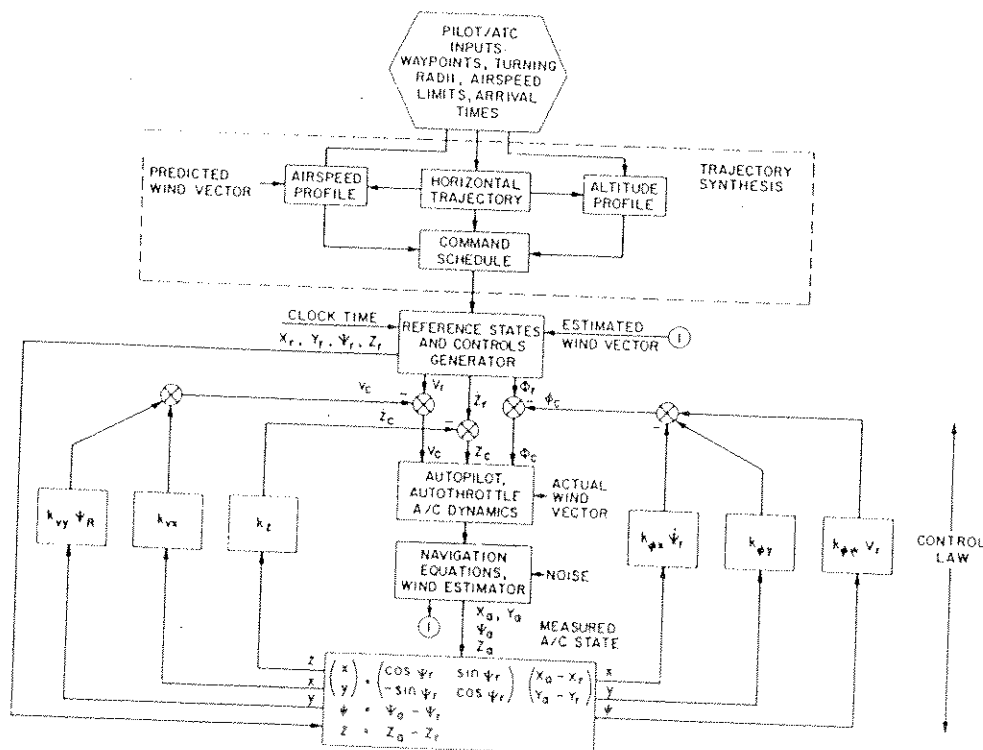


Fig. 8. Block diagram of 4-D guidance system simulation.

is to compute the perturbed quantities x , y , z , ψ , which are obtained in the transformations given in the bottom block. These quantities are multiplied by the appropriate gains and are added as required to form the perturbation controls v_c , ϕ_c , z_c . They are then subtracted from the reference controls to form the autopilot and autothrottle inputs consisting of the command bank angle ϕ_c , the command airspeed V_c and the command altitude rate \dot{z}_c .

A simplified model of the autopilot, autothrottle and aircraft dynamics was developed especially for use in 4-D guidance and air traffic control simulation studies. A detailed flow chart for this model is given in Ref. 7. The model consists of a 10th order dynamic system with hard limits on roll, roll rate, airspeed, airspeed acceleration, flight path angle and flight path angle rate. The actual wind vector is also an input to the model. Output quantities are the actual aircraft states. From these the navigation system simulation obtains the measured aircraft states and the estimated wind vector.

A complete analysis of simulation results cannot be given within the length of this paper. We shall show only the response of the control law to track a STOL type reference trajectory in the horizontal plane consisting of a 360° circular segment with radius 1220 ft and airspeed of 135 ft/sec. The reference trajectory, which has a duration of 56.7 sec. is generated with two final heading waypoints located on the circle as shown in Fig. 9. This reference trajectory is a severe test of the control law since to fly it requires a reference bank angle of 25°, almost equal to the bank angle limit of 30° used in the aircraft simulation; leaving little bank margin for nulling out errors. Figure 9 also shows the trajectories of the simulated aircraft for two initial conditions and an error in the wind estimate. The position of the reference aircraft and of the simulated aircraft is marked every 10 sec along the trajectories.

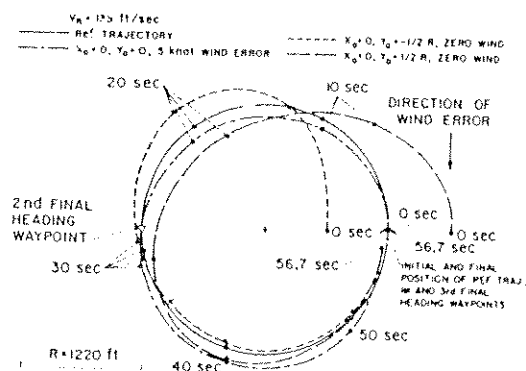


Fig. 9. Simulated flight along circular reference trajectory with initial condition errors and wind estimate error.

Starting from the two initial conditions, the simulated aircraft locks onto the reference trajectory after 30 sec of flight even though a period of bank angle limiting occurs (not shown in Fig. 9) while the control law nulls the errors. To evaluate the effect of wind estimate errors on tracking accuracy, a 8.45 ft/sec (5 knots) constant wind error was introduced. Normally, a wind estimator, which is part of the navigation system, would observe this error to a degree and refine its estimate, but in this case the estimator was disabled. The

resulting tracking error is 180 ft at the end of the trajectory, indicating the importance of accurate wind estimates in precision aircraft control.

Sequencing and Spacing Algorithm for 4-D Guidance

As described earlier, the sequencing and spacing of aircraft with 4-D guidance is achieved through the assignment of arrival times at selected points of each aircraft's terminal area approach route. The specification of the approach route and the assignment of arrival times is the responsibility of air traffic control. Only an abbreviated description of the algorithm for arrival time assignment is possible within the scope of this paper.

As each aircraft arrives at one of the feeder fixes shown in Fig. 1, the algorithm blocks off a time interval, or equivalently a distance interval around the nominal aircraft position. This interval, which moves along the trajectory with the speed of the aircraft, is checked for intersection with blocked intervals corresponding to previously scheduled aircraft. If an intersection occurs at any time prior to touchdown, the rate of movement of the block is modified by reassigning arrival times at the gate and at touchdown. If a nonintersecting interval of time cannot be found by changing the arrival times within the minimum and maximum arrival time limits, additional delay is introduced by holding and/or path stretching at the feeder fix. The time propagation of a blocked interval and its motion relative to other blocked intervals is studied in Ref. 5 by plotting distance to touchdown vs. time to touchdown. Intersecting intervals are easily spotted with the help of such time-distance diagrams.

Aircraft scheduling by means of this algorithm is currently being investigated in a real time ATC simulation. In this simulation, controller is presented with computer-generated time-distance diagrams on a CRT display in addition to the standard horizontal situation display. Inspection of the display permits the controller to pick nonconflicting arrival times manually or to monitor the automatic assignment of arrival times by the scheduling algorithm.

Conclusion

The chief advantage of the approach to 4-D guidance described here is the ability to specify and compute complex trajectories in flight. This is a highly desirable feature from the pilot's viewpoint. Another advantage is that the technique is not strongly dependent on the aircraft type, since the only aircraft parameters used in synthesizing the trajectories are performance limitations, which are treated as parameters. Furthermore, the guidance technique can be integrated with a ground based scheduling technique to form a complete air traffic control system. The precision of trajectory control and arrival time achieved with the system provides a solid base for reducing separation requirements and increasing landing rates in future air traffic control system.

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